High-Frequency Oscillation

If the frequency of oscillation is high enough, Eqs. (6) reduce to

$$i\omega B_y = B_{yy} + Q$$
$$i\omega Q = (1/\sigma)Q_{yy}$$

from which one easily obtains

$$Q = G_0 e^{-(i\omega\sigma)^{1/2}y}$$
$$B = [G_0/i\omega(\sigma - 1)][e^{-(i\omega)^{1/2}y} - e^{-(i\omega\sigma)^{1/2}y}]$$

which is of the "shear-wave" type, predicting a phase advance of 45° in the local rate-of-heat-transfer fluctuations and an equivalent phase lag in the skin-friction oscillations.

For sufficiently small values of ω, only the first term of the series expansion will be significantly important. It easily is verified that $M_1 = F + \eta F'$ and $R_1 = \theta + \frac{1}{4}\eta\theta'$. It then remains to determine N_1 and S_1. As a preliminary step, N_1 and S_1 were determined using the von Kármán-Pohlhausen method.[5] The results indicate that there exists a critical frequency ω_0 such that $x_1 = x\omega^2/G_0 = 0.7$, which separates the regions of applicability of low- and high-frequency solutions. However, to predict the results more accurately, these equations are being integrated numerically, and the results will be presented in a separate paper.

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Structural Damping

R. H. Scanlan* and A. Mendelson†

Case Institute of Technology, Cleveland, Ohio

ARE certain mathematical models of structural damping physically unrealizable? It is probable that true damping mechanisms in structures are of a quite complicated character. However, in a good many practical cases, it has appeared possible to account in a reasonable measure for the damping in an overall way by means of a linear model. It goes without saying that this model must be free of gross anomalies; a poor model may confuse the situation. When the simplified model has been chosen, nothing other than an approximation to the overall effect of the damping is expressed concerning the true damping mechanism. To inquire, then, into whether one or another simplified model itself is physically realizable or unrealizable would appear to be a less rewarding side of the question. Crandall, in a recent publication[1] raises the question of the physical realizability of one of the most familiar linear damping models; the present note discusses this model.

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* Professor of Engineering Mechanics, Engineering Division.

† On leave from NASA Lewis Research Center, Cleveland, Ohio.

The damping model in question was employed in 1938 by Theodorsen and Garrick[2] in early flutter studies, and they ascribed it to Becker and Föppl. Specifically, it employs the device of introducing linear structural damping into the typical flutter equations having complex coefficients, not as a viscous velocity term, but as a term $ig\omega_n^2 x$, where $i = (-1)^{1/2}$, $g > 0$ is a damping coefficient, and x is displacement of a typical degree of freedom, the natural circular frequency of which is ω_n. This device has the well-known effect of creating a damping term in phase with velocity $\dot{x}$ and proportional to a displacement x which has the form $e^{i\omega t}$ with $\omega > 0$, since the term is advanced 90° in phase by the factor i. Note that, in this sense, since the usual flutter analysis of the type alluded to was made precisely for nondecaying sinusoidal oscillations of the form $e^{i\omega t}$ only, the device is effective in its intended purpose within this context. Therein, incidentally, no occasion arises to consider ω other than positive.

As an equation typical of this situation, consider the following:

$$\ddot{x} + (1 + ig)\omega_n^2 x = Ae^{i\omega t} \tag{1}$$

where ω_n is the natural circular frequency of the undamped system and $\omega > 0$ is the forcing circular frequency, A being some complex constant.

Briefly recall here the solution of Eq. (1): it consists of the free vibration (solution for $A = 0$) plus a forced vibration at circular frequency ω. Crandall[1] raises the question of the physical unrealizability of the solution of Eq. (1) "if . . . negative frequencies are to be considered . . . ." Normally, as was pointed out, only positive ω is considered in the forced vibration. "Negative frequencies" are to be considered in Eq. (1), therefore, only in the cases where the situation described by Eq. (1) is extended to other meanings than the one originally intended (such as modification to greater generality of the right-hand side and, in particular, an extension into the negative Fourier domain) or in the homogeneous case, i.e., the free vibration. In examining the free vibration an anomaly is, in fact, encountered. It is found that, for *arbitrary* initial conditions, and for either $g > 0$ *or* $g < 0$, *no decaying solution exists*, but rather there *always* exists a portion of the solution having exponentially increasing amplitude with time. This is demonstrated as follows: Let g be positive *or* negative. Take $A = 0$ in Eq. (1) and assume a solution in the form

$$x = x_0 e^{i\omega_d t}$$

with $\omega_d = \omega_r + i\omega_i$ (ω_r, ω_i real). Use of this solution in Eq. (1) yields

$$-\omega_d^2 + \omega_n^2(1 + ig) = 0$$

which in turn gives the following solutions for ω_r and ω_i:

$$\omega_r = \pm\, \omega_n \left[\frac{1 + (1 + g^2)^{1/2}}{2}\right]^{1/2}$$

$$\omega_i = \pm\, \omega_n \frac{g}{\{2[1 + (1 + g^2)^{1/2}]\}^{1/2}}$$

The solution for x is then

$$x = x_{01}e^{i\omega_1 t} + x_{02}e^{i\omega_2 t}$$

with x_{01} and x_{02} arbitrary constants and

$$\left.\begin{matrix}\omega_1\\ \omega_2\end{matrix}\right\} = \pm\, \omega_n \left\{\left[\frac{1 + (1 + g^2)^{1/2}}{2}\right]^{1/2} + i\,\frac{g}{\{2[1 + (1 + g^2)^{1/2}]\}^{1/2}}\right\}$$

If one insists that the solution must represent a decaying motion for arbitrary initial conditions (x_{01}, $x_{02} \pm 0$), the sign of g must coincide with that placed before ω_n and therefore will be different for each of the two terms of the solution, which is in contradiction to the original assumption. Thus it is seen that the fact of whether oscillations are decaying or diverging is not governed, in this equation, by a single sign for g. For $g > 0$, for example, oscillations always will diverge unless x_{02} is taken equal to zero, which then leaves only one arbitrary constant to satisfy two initial conditions; further, the condition $x_0 = 0$ cannot be satisfied without reduction to a trivial solution. Among other things, the situation described points out an error in Ref. 4 which was indicated to the authors by Crandall.[3]

The fact that the physical system is clearly a decaying one for positive damping makes it appear desirable to attempt to remove the anomaly just mentioned. Crandall[1,3] suggests a method of accomplishing this, the two alternatives of which, however, must be distinguished from each other. In Ref. 1, he suggests replacing $1 + ig$ by $1 + ig \operatorname{sgn}\omega$. This applies to the forced case only. The alternative method (free case) is to replace $1 + ig$ by $1 + ig \operatorname{sgn}\omega_r$. The second alternative does, in fact, assure a decaying solution for the homogeneous part of Eq. (1). However, it does not retain damping in phase with velocity. The $1 + ig$ device is, strictly speaking, correct only for the originally intended use, namely, simple harmonic motion of the form $e^{i\omega t}$, $\omega > 0$; that is, it represents damping proportional to displacement and in phase with velocity only in this case. In any other case, such as an oscillation $e^{i\omega_d t}$ with ω_d complex, for example, the velocity, $i\omega_d x$ itself clearly has components both in and 90° out of phase with $ig\omega_n^2 x$.

The use of the first alternative, $1 + ig \operatorname{sgn}\omega$, yields absurd results[1] for the response in the case where the right-hand side of Eq. (1) is replaced by a pulse and the system is analyzed employing the Fourier transform over $-\infty \leq \omega \leq \infty$. The particular absurdity that occurs when the Fourier transform is employed is a result of the artificial discontinuity introduced in g at $\omega = 0$.

In brief, therefore, the factor $1 + ig$ $(g > 0)$ works to give the desired damping effect only in the sinusoidal case intended and not in others. A modification of it is unnecessary in the original (sinusoidal) case; one suggested modification fails to provide damping proportional to displacement and in phase with velocity in the free case and exhibits other anomalies under Fourier transform treatment in nonsinusoidal forced cases. If damping proportional to displacement and in phase with velocity is desired, which will eliminate the previous anomalies in the free case (perhaps as a tour de force), the authors suggest the following:

Suppose that one is given a single degree of freedom system with undamped natural frequency $\omega_n = (k/m)^{1/2}$ and with a damping force having the properties that it is 1) proportional to the displacement, and 2) in phase with the velocity. Then the free oscillations of such a system can be described exactly by the following differential equation:

$$m\ddot{x} + (gk/\omega_d)\dot{x} + (1 + g^2)^{1/2}kx = 0 \qquad (2)$$

where the damped frequency ω_d is real and is given by

$$\omega_d = \omega_n\{[1 + (1 + g^2)^{1/2}]/2\}^{1/2} \qquad (3)$$

A similar formulation is given by Pinsker[5] in commenting on a paper by Soroka.[6] It can be determined readily that Eq. (2) satisfies the desired requirements precisely and does not have any of the anomalies associated with Eq. (1). It therefore provides an exact viscous model corresponding to the desired structural damping model.

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Effects of Mass Addition on the Stability of Slender Cones at Hypersonic Speeds

CLARENCE A. SYVERTSON* AND JOHN B. MCDEVITT*
NASA Ames Research Center,
Moffett Field, Calif.

THE effect of mass addition on the flow over bodies moving at hypersonic speeds has been studied by several investigators (e.g., Cresci and Libby[1]). In most of this work, primary attention logically has been directed toward the effects of foreign-gas injection on heat transfer and pressure distributions, and, principally for this reason, most of the work has been done at zero angle of attack. The foreign gas can be provided either by some active injection system or by the action of an ablation heat shield. With increasing rates of injection, the basic flow about the body can be affected significantly. One such effect was observed in the paper by Cresci and Libby,[1] where it was shown that the shock-wave standoff distance can be increased by gas injection at the nose of a body.

Another effect of mass addition at the nose has been investigated in the 14-in. helium tunnel at the NASA Ames Research Center. In this study, a cone having a semivertex angle of 10° was tested at a Mach number of 21 and at a Reynolds number (based on length) of 4.5×10^6. The cone was 2 in. in diameter at the base and had a hemispherical tip of 0.076-in. radius. In this hemispherical tip were one hole 0.040 in. in diameter and eight holes 0.025 in. in diameter. From these holes helium was injected at various rates, and the effects of this gas injection on forces and moments were determined at angles of attack up to about 14°.

Some of the results of this investigation are shown in Fig. 1. For these results, the mass rate parameter $\dot{m}$ is the ratio of the mass rate of injection to the product of the freestream velocity, freestream density, and body base area. These results show that the mass addition decreases the stability at low angles of attack and increases it at intermediate angles. In fact, a crossover occurs, and at higher angles the pitching moments are increased in magnitude compared to those for the body without injection. With increasing mass addition, the changes in stability and moments become more pronounced, and the crossover angle of attack increases. The normal force, however, decreases with increasing injection rate at all angles of attack, although the reductions are somewhat less at the higher angles. A possible explanation for this behavior can be obtained with the aid of the results of another series of tests. The same cone with a series of oversized spheres at the tip was tested at a Mach number of 18 and a Reynolds number of 3.7×10^6. The direct contribution of the sphere drag on the normal forces and pitching moments was assumed to correspond to a drag coefficient of

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* Research Scientist.

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